Recall basic Cons. of Energy...

$$\rho c\left(\frac{\partial T}{\partial t} + \underline{v}\cdot\nabla T\right) = \underbrace{k\nabla^2 T}_{\text{or } \nabla\cdot(k\nabla T)} + \dot{U}'''_{gen}$$

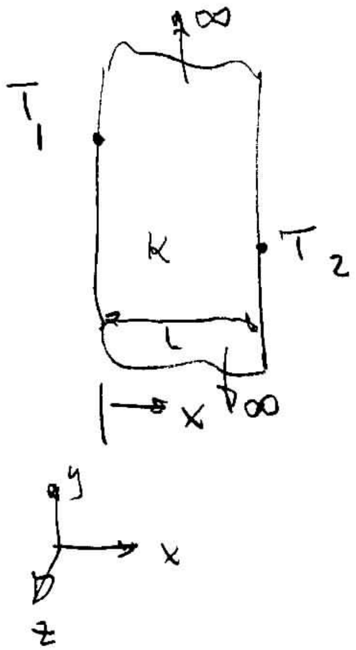
Assumes $\rho, c, k = \text{¢}$

For S.S., $\underline{V} = \underline{0}$, $\dot{U}'''_{gen} = 0$ ... $k\nabla^2 T = 0$

Now pick your coordinate system.

Cartesian $$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

- For a 1-D wall of thickness L



What is $T(x)$?

Note: T is assumed to only vary in x-direction

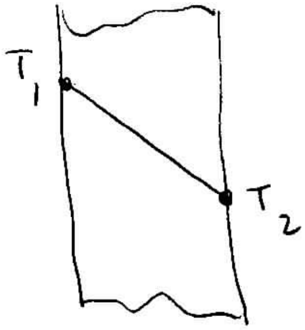
$$\frac{d^2T}{dx^2} = 0$$ ∫ once...

$$\frac{dT}{dx} = \text{¢}_1$$ ∫ once

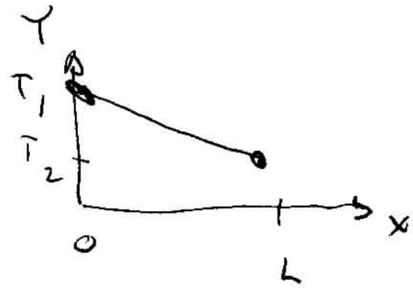
$$T = \text{¢}_1 x + \text{¢}_2$$

Use B.C. $$T\big|_{x=0} = \text{¢}_2 = T_1$$

$$T\big|_{x=L} = \text{¢}_1 L + T_1 = T_2 \Rightarrow \text{¢}_1 = (T_2 - T_1)/L$$



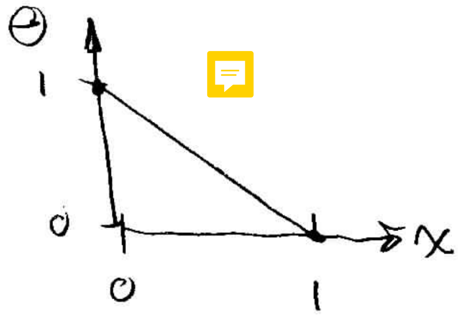
$$T = \frac{(T_2 - T_1)}{L} x + T_1$$



or

$$\underbrace{\frac{T - T_1}{T_2 - T_1}}_{\equiv \Theta} = \underbrace{\frac{x}{L}}_{\equiv \mathcal{X}}$$

$$\boxed{\Theta = \mathcal{X}}$$



But recall

$$q = -kA \frac{\partial T}{\partial n} = -kA \frac{dT}{dx}$$

$$= -kA \frac{(T_2 - T_1)}{L}$$

$$= \left(\frac{kA}{L}\right)(T_2 - T_1)$$

Compare

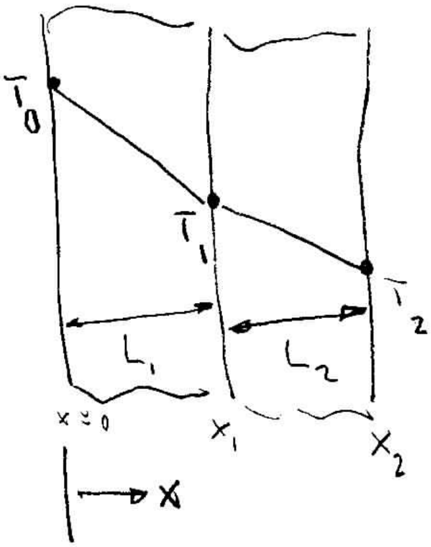
$V = IR$

$\Delta T = q R_{therm}$

or $$q = \frac{(T_2 - T_1)}{R_{therm}}$$

so $\boxed{R = L/kA}$ for wall

• How about 2 walls touching...



$$q = \frac{T_1 - T_0}{R_1} \qquad q = \frac{T_2 - T_1}{R_2}$$

$$T_1 - T_0 = q R_1 \qquad T_2 - T_1 = q R_2$$

Add these equations

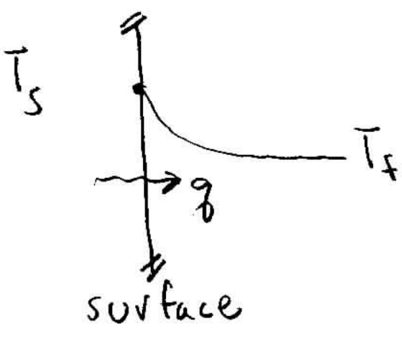
$$(T_1 - T_0) + (T_2 - T_1) = q R_1 + q R_2$$

$$T_2 - T_0 = q(R_1 + R_2)$$

$$\boxed{\Delta T_{overall} = q \sum R_i}$$

• How about 2 touching wall, each of which is in contact with a moving fluid...

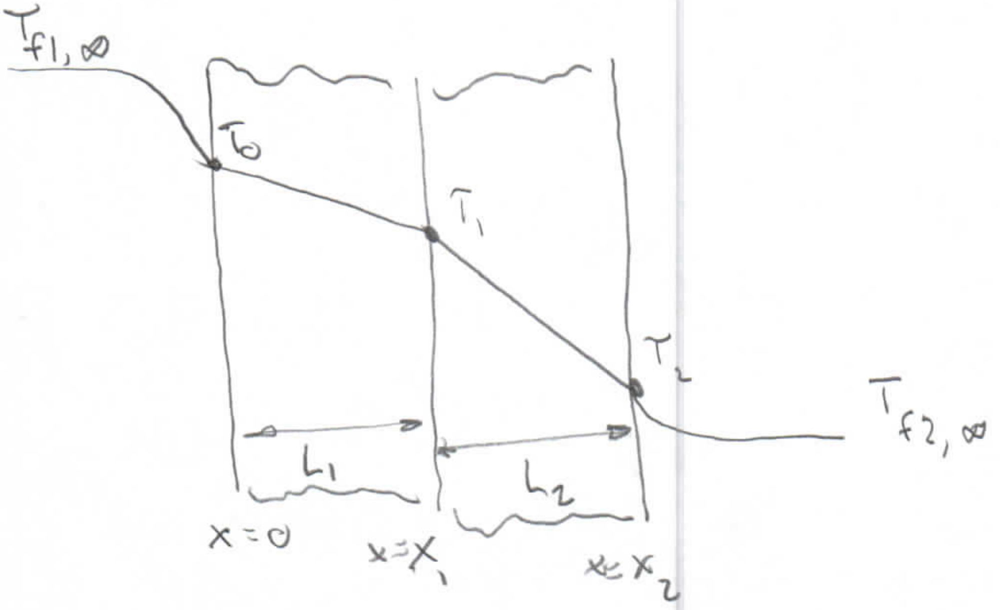
First recall



fluid conv. heat transfer coef.

$$q = hA(T_s - T_f) = \frac{T_s - T_f}{R_{conv}}$$

$$\Rightarrow R_{conv} = \frac{1}{hA}$$



$$q = \frac{T_0 - T_{1,\infty}}{R_{conv,1}}, \quad \frac{T_1 - T_0}{R_1}, \quad \frac{T_2 - T_1}{R_2}, \quad \frac{T_{2,\infty} - T_2}{R_{conv,2}}$$

so

$$T_0 - T_{1,\infty} = R_{conv,1}\, q \qquad T_1 - T_0 = q R_1 \qquad T_2 - T_1 = q R_2 \qquad T_{2,\infty} - T_2 = q R_{conv,2}$$

Add these

$$(T_0 - T_{1,\infty}) + (T_1 - T_0) + (T_2 - T_1) + (T_{2,\infty} - T_2) = q\left(R_{conv,1} + R_1 + R_2 + R_{conv,2}\right)$$

$$\underbrace{T_{2,\infty} - T_{1,\infty}}_{\Delta T_{overall}} = q \sum R_i$$

where
$$R_{total} = \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A}$$

$$= \frac{1}{A}\underbrace{\left(\frac{1}{h_1} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h_2}\right)}_{\text{call this } 1/U} \equiv \frac{1}{AU}$$

(5)

So finally

$$T_{2,\infty} - T_{1,\infty} = q\left(\frac{1}{AU}\right)$$

$$\Delta T_{overall} = \frac{q}{A\, U_{overall}}$$

$$q = A\, U_{overall}\, \Delta T_{overall}$$

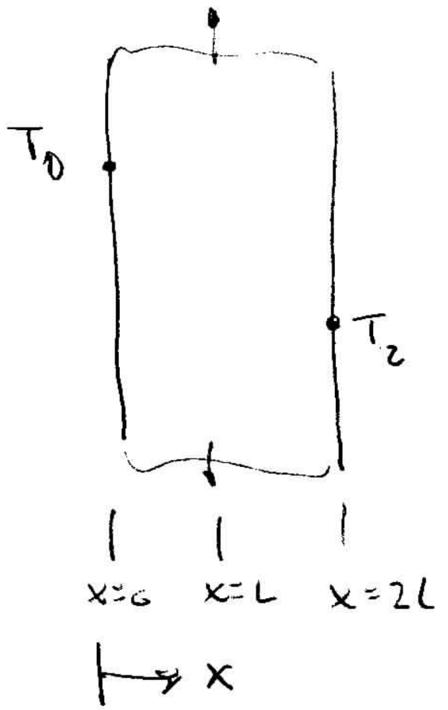
$$\frac{1}{U_{overall}} = \frac{1}{h_1} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h_2}$$

Now, 1-D planar, S.S., ¢ matl. prop, with heat generation

$$\rho c\left(\frac{\partial T}{\partial t} + \underline{v}\circ\nabla T\right) = k\nabla^2 T + q'''$$

$\vdots$

$$0 = k\frac{d^2T}{dx^2} + q''' \Rightarrow k\frac{d^2T}{dx^2} + q'''_{generation}$$



$$\frac{d^2T}{dx^2} = \frac{-q'''}{k}$$

$$\frac{dT}{dx} = -\frac{q'''}{k}x + ¢_1$$

$$T = \frac{-q'''}{2k}x^2 + ¢_1 x + ¢_2$$

B.C. $T\big|_{x=0} = T_0$

$T\big|_{x=2L} = T_2$

– why not T_1?

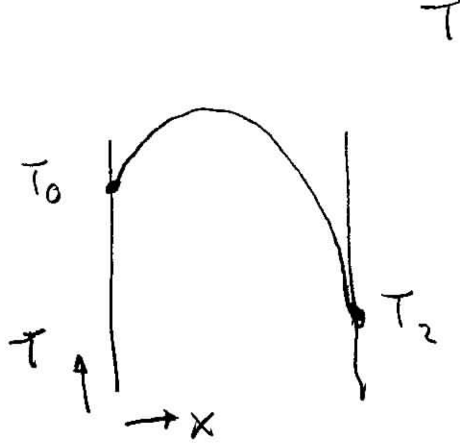
Use B.C.

$$T\big|_{x=0} = ¢_2 = T_0$$

$$T\big|_{x=2L} = \frac{-q}{2k}4L^2 + ¢_1 2L + ¢_2$$

leads to

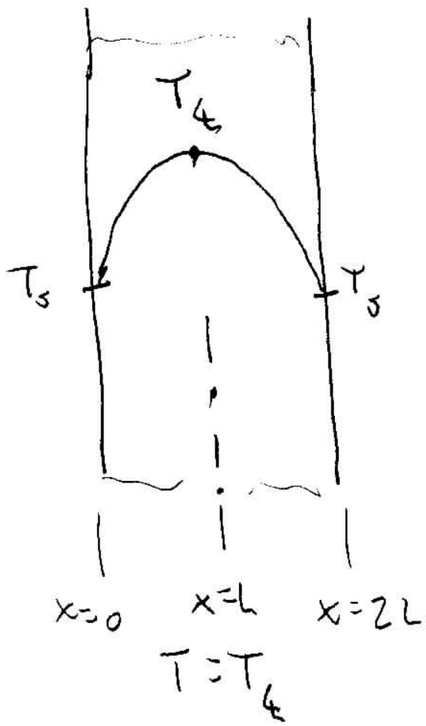
$$T = \left[\frac{T_2 - T_0}{2L} + \frac{q'''}{2k}(2L - x)\right]x + T_0$$



- You can do the algebra to calculate

$$q = -kA\frac{dT}{dx} \quad \text{at each surface (left + right)}$$

- A nice special case is if $T_2 = T_0 \equiv T_s$



$$T = \left(\frac{q'''}{2k}\right)(2L - x)x + T_s$$

What is $T_\text{¢} = T\big|_{x=L}$

$$= \frac{q'''L^2}{2k} + T_s$$

or $$T_\text{¢} - T_s = \frac{q'''L^2}{2k}$$

so then the T-dist is

$$T - T_s = (T_\text{¢} - T_s)\left(2 - \frac{x}{L}\right)\left(\frac{x}{L}\right)$$

or

$$\frac{T - T_s}{T_\text{¢} - T_s} = \left(2 - \frac{x}{L}\right)\left(\frac{x}{L}\right)$$

if $\theta = \frac{T-T_s}{T_\text{¢}-T_s}$ and $\chi = \frac{x}{L}$ we get

$$\boxed{\Theta = (2-\chi)\chi}$$